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TOWARDS A NEW SYSTEM FOR ALLOCATING THE
COST OF CAPACITY

by E. B. Berman, Consultant*
Cost-Analysis Department
The RAND Corporation

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*Operations Evaluation Group, Navy Department

TOWARDS A NEW SYSTEM FOR ALLOCATING THE COST OF CAPACITY

In this paper a concept of capacity cost is presented. In this new
capacity cost system a requirement for an additional unit of capacity does
not lead either to no cost if the unit is already available or to the full
cost of building an additional unit if it is not already available. The
former event implies a cost because it advances the time when we should
expect to construct an additional unit of capacity. The latter event
offers a savings to offset the full cost of constructing a unit of capacity.
in the form of a probability that the unit we construct now would have
been needed for the next generation anyways.

We use the term "capacity" in a functional sense; thus we include in
capacity, facilities, support equipment, support organizations, and
personnel which have significant initial costs and to which the concept
of capacity therefore appertains. For our analysis let us assume that we
are comparing two weapon systems which are identical except that one
weapon system uses a base of type A and the other uses a base of type B.
We therefore want to assess the true cost to the Air Force of requiring
an additional unit of each type of base. We shall carry forth an analysis
for hypothetical base type A, but of course all of the methodology could
apply to any other kind of capacity.

Let us start with the case of certainty in which we can predict with
accuracy what the requirement for bases of type A will be for an indefinite
period into the future. Figure 1 represents the (known) requirements for
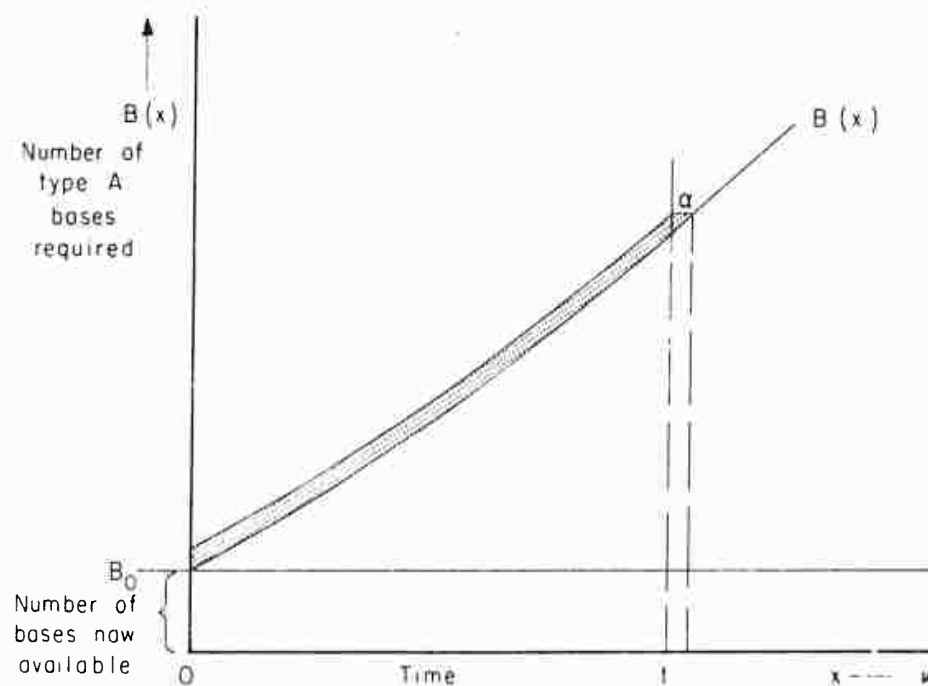bases of type A over time.

Fig. I —— Hypothetical base requirement time schedule

The shaded area in Figure 1 represents an additional unit of Base type A required for t years. It seems fairly clear that under the base schedule pictured in Figure 1 the incremental base used between years 0 and t has not caused an additional base to be built; it has however advanced to time period 0 the construction of a base which could otherwise have been postponed to period t (more accurately to period $t + \frac{t}{2}$). Thus the cost of requiring an additional base for t years tends to be in the nature of an interest charge if we can predict that the base will be needed after it is released by the subject weapon system in year t.

The base will cost just exactly what it would have cost if we were able to float a loan for it and at the end of t years, sell the base to another weapon system and repay the loan. The only cost in that case would be the interest payment on the loan for t years. In exactly this way, we borrow the base for t years and pay interest on it.

The cost is not too different if the Air Force currently has a surplus of type A bases as long as that surplus will be used up before year t. Figure 2 illustrates the additional requirement for a type A base starting with a surplus of type A bases.
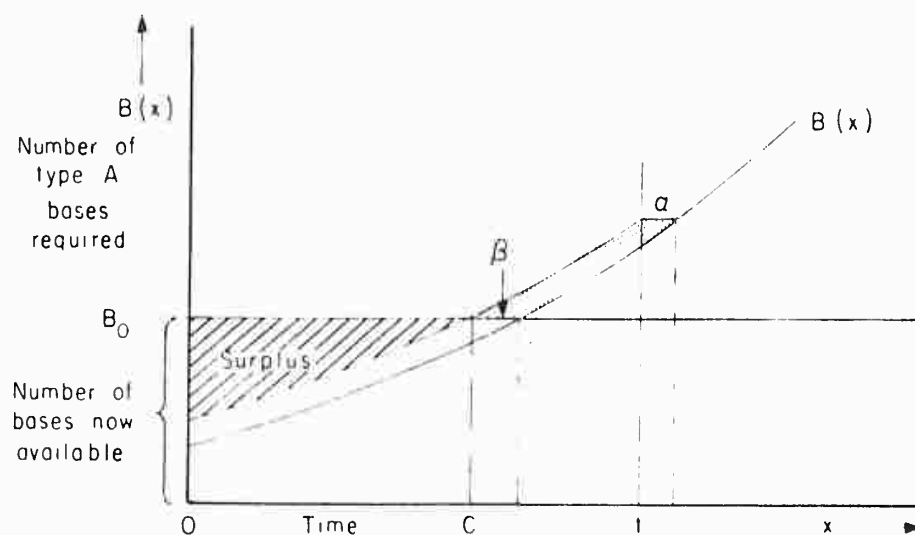

Fig. 2 — Hypothetical base requirement time schedule
The initial surplus

In the Figure 2 case, the additional base costs us an interest charge from year C to year t, or more accurately from year $c + \frac{\beta}{2}$ to year $t + \frac{\lambda}{2}$. This is equal to an interest charge for $(t - c)$ years discounted back from year C to year O. Here we borrow a base in year $C + \frac{\beta}{2}$ and repay it in year $t + \frac{\lambda}{2}$ and the true cost of the base is the cost of floating a loan in year $C + \frac{\beta}{2}$ and repaying it in year $t + \frac{\lambda}{2}$. It goes without saying that if C is later in time than t there is properly no charge for the use of the base.

If the base that is now required will have no use after its time of release (time period t) then the cost of the base is as if the base were needed for an infinite period of time. Figure 3 illustrates a base requirement time schedule in which the base released at time t is never required again.
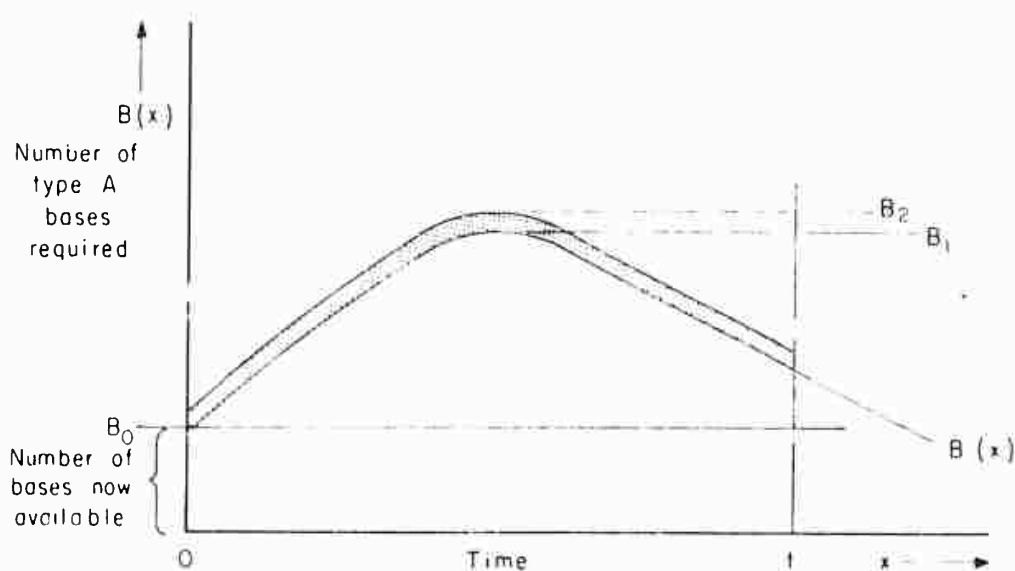
Fig. 3 — Hypothetical base requirement time schedule.
Case of declining requirement at time t

In Figure 3, the incremental requirement caused by the subject weapon system (shown as the shaded area) has raised the peak requirement for bases from B_1 to B_2. It is therefore proper to assess the cost of one whole base against the subject weapon system. The analysis is similar when the Air Force begins with a surplus of bases as long as that surplus is exhausted by time t. This case is illustrated in Figure 4.
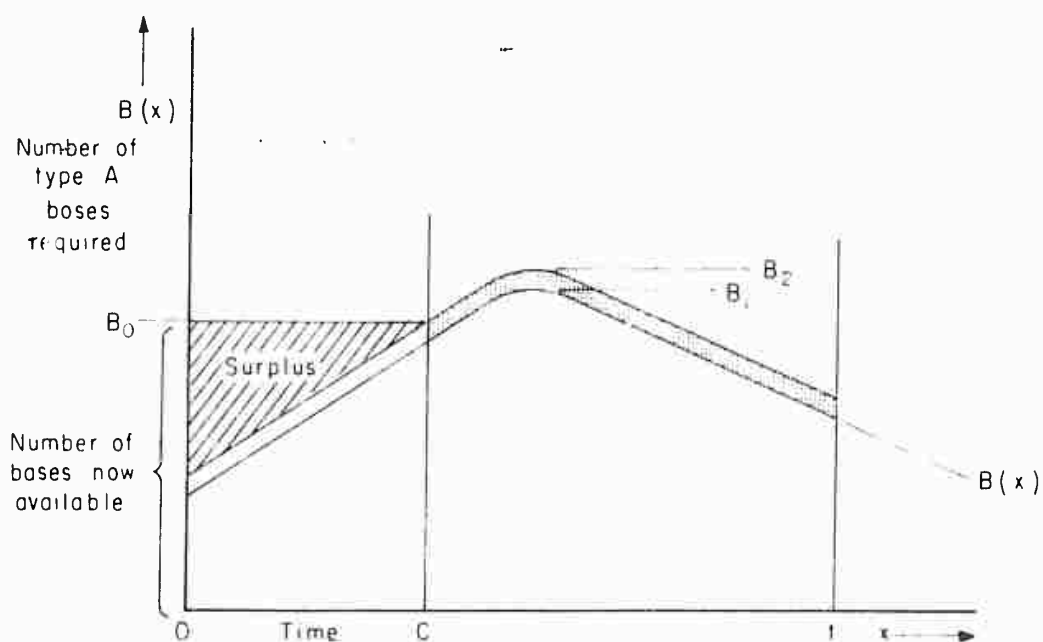

Fig. 4 —— Hypothetical base requirement time schedule:
Case of initial surplus with declining
requirement at time t

The case illustrated in Figure 4 is similar to the case of Figure 3.
Here no additional base requirement is caused until year C, but once the
base is required, the requirement lasts forever; as in Figure 3, the peak
requirement is raised from B_1 to B_2. We therefore properly charge a
whole base to the subject weapon system. Since however the additional
base will not be built until year C, we discount the cost of the base back
from year C to year 0 with an appropriate interest rate to obtain the
appropriate year 0 cost. If B_2 is less than or equal to B_0,
there should properly be no charge to the weapon system for the use of
the base.

An even more interesting case is the requirement schedule which is
falling at time t, but later rises so that the base released at time
t will have some use at some time in the future. This case is illustrated
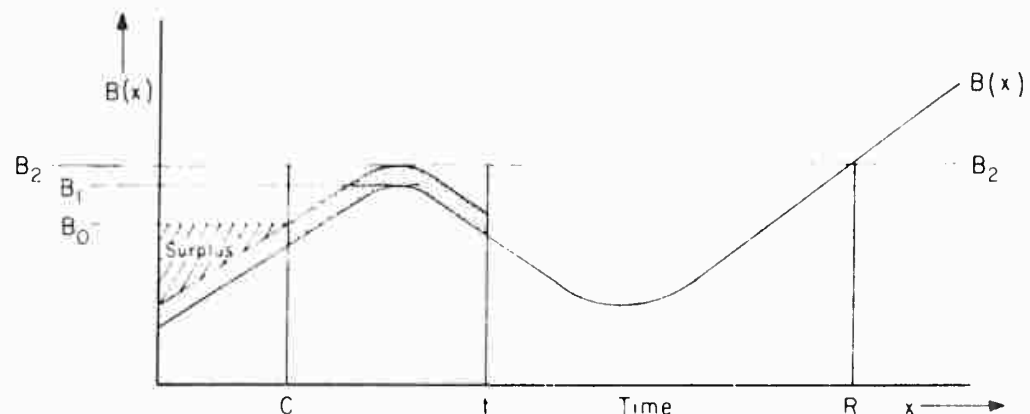in Figure 5 under the assumption of an initial surplus of type A bases.

Fig. 5 — Hypothetical base requirement time schedule:
Case of initial surplus with requirements de-
clining at time t and later rising

Under the time schedule illustrated in Figure 5, the incremental base requirement we now impose (the shaded area) causes an extra base to be built in year C; the extra base stays in surplus between year t, when it is released by the subject weapon system, and year R, when rising requirements absorb the surplus of bases. The appropriate present cost is an interest charge for (R - C) years discounted back to year 0. If there is no initial surplus, year C equals year 0.

Overhead Costing under Uncertain Requirements Schedules

Let us now assume that the schedule of requirements for base type A is uncertain and given only by various probability functions to be described below. We shall begin with the simplest case, that of the requirement for one base over an infinite period of time (forever). This case is relatively simple because it includes only one kind of uncertainty, the timing of the construction of a new base. The requirement for one base over a finite period of time has the added uncertainty in the timing of the need for the base after its release by the subject weapon system.

There is no possible uncertainty in the requirement forever of a base not in surplus; it is needed now for certain and forever. If a surplus exists however we may allow uncertainty in when or if the construction of the additional base will occur. Let us introduce some terminology needed for this case.

$f(x)$ = the probability that the surplus of bases will be used up and a new base built in year x (see Figure 2, x = c). The sum of $f(x)$ is $\int_{0}^{\infty} f(x)dx$; this sum may be less than or equal to 1. If it is less than 1, the difference represents the probability that an additional base will never be built.

d   =   a discount factor

   d_j^i =   a discount factor between years i and j at an appropriate
           interest rate.

K   =   the cost of building a base of type A.

C   =   the present cost of requiring a base of type A.

We may now write the following equation for C, the present cost of requiring
a base of type A, assuming that the requirement is to last forever and that
the bases are now in surplus.

(1.) $C = K \displaystyle\int_0^\infty d_x^o \, f(x) dx.$

In this equation d_x^o , the discount factor between the present

(year o) and year x, is multiplied by f(x), the probability that the
incremental base we have caused to be built will be built in year x. The
cross products are summed over all values of x by the integral to obtain
an expected discount factor which is then multiplied by K, the cost of
the base.

A somewhat more complicated case under uncertainty is the requirement
for a finite period of time with no initial surplus. There are two
elements of uncertainty in this case; (1) the level (although not the
timing) of the peak requirement preceding the year of release (year t in
Figure (5) and (2) the year after t in which that peak would again be
reached, if ever (year R in Figure 5). Let us introduce additional
terminology before formulating this second case.

g (y) = the probability that the peak requirement for Base type A
between year 0 and t will be y.

$$\int_{y_0}^{\infty} g(y) \, dy = 1 \text{ where } y_0 \text{ is the requirement for bases in year 0; in other words, there is no probability that the peak be less than } y_0.$$

$\emptyset_y$ (x) = the probability that a requirement level of y will first
be reached or exceeded in year x ($\emptyset_y(x)$ is defined from

t to ∞ .) $\int_{t}^{\infty} \emptyset_y (x) dx \leq 1.$

We may now write the following equation for C, the present cost of
requiring a base of type A for t years, assuming that there is no initial
surplus of bases:

$$(2.) \quad C = K \left[1 - \int_{y=y_0}^{\infty} \int_{x=t}^{\infty} d_x^0 \, \emptyset_y (x) dx \, g(y) dy. \right]$$

In this formulation, the whole cost of the base is charged (K) and
then a refund is provided in the form of the double integral term represent-
ing (1) the probability that the base made available in year t will be
put to use in year x (x > t), and (2) the discounted present value of the
base in year x.

The factor d_x^o is the percent of present cost represented by the value of the refunded base in year x; this is multiplied by $\phi_y(x)$, the probability that the refunded base will first be put to use in year x if the pre-release peak requirement is y; the cross products are then summed by the inner integral:

$$\int_{x=t}^{\infty} \ldots\ldots dx$$

over all values of x from the year of refund, t, to infinity, to obtain an expected value of the discount factor d_x^o at the given level of y. This expected value is then multiplied by the probability of that level of y occurring, $g(y)$, and summed by the outer integral:

$$\int_{y=y_0}^{\infty} \ldots\ldots dy,$$

over all values of y from y_0 to infinity.

The most difficult case of all under uncertainty is the case of a base required for a finite period in which there is an initial surplus of the base. Here there are three elements of uncertainty: (1) the level of the peak preceding the year of release; (2) the year after t in which that peak would again be reached, if ever and (3) the year preceding release in which the initial surplus would be exhausted, if at all. The third element starts the time clock of cost going, if the surplus is used up; the first and second elements determine if and when the time clock will stop.

We may now write the following equation for c, the present cost of requiring a base of type A for t years, assuming that there is an initial surplus of type A bases:

$$(3) \quad C = K \left[\int_{z=0}^{t} d_z^o f(z)dz - \int_{y=B_o}^{\infty} \int_{x=t}^{\infty} d_x^o \; \emptyset_y(x)dx \; g(y)dy \right], \quad \text{where} \quad B_o$$

is the quantity of type A bases initially available.

The first term represents the expected cost of building a base. It is similar to equation 1 with the single exception that the upper limit of the integral is t rather than ∞ representing the fact that the use of the base is free if the surplus is not used up by year t.

The second term represents the value of refunding the base in year t. It is the same as equation 2 except for the lower limit on the y integral. The use of B_o as the lower limit represents the fact that a peak pre-release requirement (y) less than B_o implies that the surplus was not used up, hence there is no cost and also no possible refund. Note that the total probability of the surplus being used up by year t:

$$\int_{z=0}^{t} f(z)dz$$

is equal to the total probability of a pre-release peak greater than or equal to B_o:

$$\int_{y=B_o}^{\infty} g(y)dy.$$

Alternatively it may be stated that the probability of the surplus not being used up by year t:

$$1 - \int_{z=0}^{t} f(z)dz$$

is equal to the probability of a pre-release peak between y_0 and B_0:

$$\int_{y_0}^{B_0} g(y)dy = 1 - \int_{B_0}^{\infty} g(y)dy.$$

Thus the cost and refund terms of equation 3 are balanced in the sense that a refund is possible only under conditions in which the cost would have occurred first.

MODIFICATIONS

The foregoing analysis has implicitly assumed that the bases were useable without modification by both the subject weapon system and the weapon system which occupies the released base. In the real world there are almost always modification costs when a base built for one weapon system is used for a different weapon system. Let us identify three possible modification costs as they apply to our methodology:

M_1 = the cost of modifying a base now in surplus to make it useable for the subject weapon system.

M_2 = the cost of modifying a base now in surplus to make it useable for a weapon system other than the subject weapon system.

M_3 = the cost of modifying (or remodifying) a base released by the subject weapon system to make it useable for another weapon system.

$$0 \leq M_1 < K;$$
$$0 \leq M_2 < K;$$
$$0 \leq M_3 < K.$$

Equations 1-3 may now be rewritten:

$$(1a) \quad C = (K-M_2) \int_0^\infty d_x^0 \, f(x) \, dx + M_1;$$

$$(2a) \quad C = K - (K-M_3) \int_{y=y_0}^{\infty} \int_{x=t}^{\infty} d_x^0 \; \emptyset_y(x)dxg(y) \; dy$$

$$\text{and } (3a) \quad C = (K-M_2) \int_{z=0}^{t} d_z^o \; f(z)d_z - (K-M_3) \int_{y=B_0}^{\infty} \int_{x=t}^{\infty} d_x^0 \; \emptyset_y(x)dx \; g(y)dy.$$

All of the preceding analysis assumes that the unit of capacity has
no scrap value. It is not possible to introduce scrap value into the
analysis formally without having first a rule for discarding surplus
capacity (e.g., any base in surplus twenty years will be sold or dismantled),
Since without such a rule it would not be possible to assess a specific
discount factor on this value. Generally however a conservative policy,
in which the unit of capacity was held in surplus many years before
discarding, would cause the scrap value to be nearly discounted out
of existence.